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ABSTRACT

Focusing on the question that is being discussed in a mathematics lesson is suggested as an approach to teaching developmental mathematics at the college level. Ways in which students can be involved with the question are described. First, some contagious cliches that are used by students are given. Then a few antidotal aphorisms and educational epigrams about learning are listed. Following these are specific exploration exercises to help develop the alternative of focusing on the question, with commentary on how to use them. They are designed to give students a successful introductory experience with each new concept. (MNS)

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FOCUS ON THE QUESTION FOR INTERMEDIATE ALGEBRA

Excerpts from presentation for AMATYC
National Convention, November 1983

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Focus on the Question for Intermediate Algebra

Excerpts from presentation for AMATYC National Convention, November 1983

Unfortunately, the teaching of developmental mathematics at the college level is too often a continuation of telling students what to do, then hoping that they will remember how to do it. If we want to change attitudes toward mathematics and improve mathematical learning, we must make pre-calculus courses truly developmental. It is especially important to do so if we want to encourage women and minorities to consider non-traditional fields. They need a more contemporary mathematical approach to prepare them for advanced levels in fields wherein they are frequently restricted to entry level positions. (How many women fail to complete graduate degrees because of mathematical deficiencies?)

The alternative is an examination of and a focus upon the question that you are discussing. If you are discussing an applied problem, emphasize what is being asked. If you are considering a difficult computation, focus upon the mathematical structure of the expression or equation. In short, when you expose students to a new mode of operation, get them involved in the process of examining questions.

Because an understanding of the question is a significant step towards reaching the answer, you should encourage requests to explain the question. Such queries as "What is being asked?", or "What are we looking for?" are usually more pertinent and indicate greater involvement than "How do we do it?".

This paper discusses ways in which students can become

involved with "the question," so that those who have been sociologically and academically discouraged from studying mathematics can find a stimulus for doing so. My experience has shown that this approach is especially successful with intellectually curious women whose academic and career aspirations were distorted to exclude the study of mathematics.

Let us look at some Contagious Cliches that helped bring about the distorted situation we have today, and then balance them with a few Antidotal Aphorisms and Educational Epigrams. This will be followed by specific Exercises that help develop an alternative view of learning mathematics. If used to their fullest, such Exercises will also stimulate a desire to study and utilize mathematics in a wide range of fields.

CONTAGIOUS CLICHES

- It's not fair to expect me to do well in math, I'm * * * * *

(* * * * * Select one or provide a write-in candidate).

a woman	a football player	an artist
Hispanic	too sensitive	politically active

- I can't learn this; it's different than the math I studied in high school.
- Math isn't something you talk about; maybe you let someone tell you about it.
- It doesn't help much to tell me that this stuff is good for me.
- Tell me what to do and I'll try it.

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ANTIDOTAL APHORISMS

+ Understand what's going on; it's less threatening that way.

+ When it's the question that's hard, use the answer; get insight from hindsight.

+ Slow success is better than fast failure.

+ "It is all right to make mistakes....It is best if it is a different mistake each time." ---Eric Johnson

+ "If you don't know where you're going, you will probably end up somewhere else." ---L. J. Peter

EDUCATIONAL EPIGRAMS

* "Let us teach guessing." ---George Polya

* It's not what you cover; it's what you uncover.

* Miskakes are hard to find.

* I hear, and I forget
  I see, and I remember
  I do, and I understand.

  ---Chinese Proverb

* If you give a man a fish,
  You feed for a him day.
  If you teach him how to fish,
  You feed him for life.

  ---wisdom of the ages

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ALGEBRA: INTRODUCTORY AND INTERMEDIAT

FOCUS ON THE QUESTION

A Point of View about Learning and Involvement

* Students should be active partners in their learning.
* A "mature student" is one who is ready to participate in the learning process.
* A remedial course should help a student reach this maturity.
* Patterns and relationships help in situations that are seemingly new.
* Checking one's solutions is essential.
* Interesting presentation must blend with an instructive narrative.
* There is a great variety among students, so a course should accommodate a range of abilities, and appeal to different modes of learning.

Comparison with Others

The content of my course in Introductory Algebra is similar to its contemporaries at this level; the difference is in the intent and its potential effect upon students. My approach--more than others--is designed to involve the reader with the process of learning, as well as with the development of skills. It aims to overcome both cognitive and affective obstacles that college students face when they study elementary, pre-college mathematics. When successful with this approach, students are more likely to continue their study of mathematics beyond the minimal requirements.

To encourage involvement, the style is personal; it is also helpful, with pertinent advice about studying and comments about mathematical insights. The student is encouraged to examine new situations, is given full solutions, and is helped to decide what to study. Frequent written Comments, Advice, etc. are included in the class notes and on test corrections, to emphasize this personalized style of assistance.

A P P E N D I X

C O M M E N T A R Y

How to Use Exploration Exercises

In these notes, Exploration Exercises and their Solutions supplement the regular worked examples that you expect to see in a text. The computations and problem solving that you do with explorations will help you focus on algebraic concepts which are necessary for future work. The following procedure is recommended to help you make a substantial contribution to your own learning.

Procedure:

(1) Read the discussion that prepares you for a set of Exploration Exercises.

(2) Copy each of the Exploration Exercises. They are based upon ideas and skills that demand attention, and copying helps you focus on the intent of the Exercise.

(3) Examine the Exercises and look for common elements. If you notice similarities, it should help you understand the reasoning behind the computations.

(4) Complete the Exercises, using any method that seems right to you, then make some guesses about shortcuts.

(5) When you finish a set of Exploration Exercises, read the Solutions. They exhibit details and provide a further explanation, including a discussion of procedural items such as (2), (3), and (4) above.

(6) Refer to previous Exploration Exercises and Examples when needed in later work.

Exploration Exercises **Using Generalizations**

See the Commentary How to Use Exploration Exercises, then explore sequences by doing the following Exercises. A detailed discussion is included with the Solutions that follow these Exercises. Solutions supplement the explanations that were given in previous Examples.

In the Exercises below, replace each # with the appropriate value of the term.

(1) Write the first four terms and the tenth term of a sequence whose nth term is given below. This Exercise gives you a chance to explore sequences whose general terms look similar. You can use line (b) to help with (c) and (d), but line (a) is significantly different.

General	1st	2nd	3rd	4th		tenth
(a) n + 10 :	#,	#,	#,	#,	...,	#
(b) 10n :	#,	#,	#,	#,	...,	#
(c) 10n - 10:	#,	#,	#,	#,	...,	#
(d) 10n + 10:	#,	#,	#,	#,	...,	#

Reminder: For the first term, substitute 1 for n. To find the 10th term, you don't need to know the 9th; let n = 10.

Note: Some general terms are a sum or difference of "terms". The word "term" is used in two different ways here; in one case it is part of a sequence, in the other it is part of a sum or difference.

(2) [See Solutions; the Exploration Exercise is not shown herein].

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Exploration Exercises **Solutions and Explanations**

Using Generalizations

(1) Given the rule for the nth term you can find any term and make a table to exhibit a pattern.

	General	1st	2nd	3rd	4th		tenth
(a)	n + 10 :	11,	12,	13,	14,	...,	20
(b)	10n :	10,	20,	30,	40,	...,	100
(c)	10n - 10:	0,	10,	20,	30,	...,	90
(d)	10n + 10:	20,	30,	40,	50,	...,	110

(2) Each part of this Exercise has the general term dn + b. For part (a) you substitute d = 5 and b = 1; for part (b) d = 1 and b = 5 ; for (c) d = 7 and b = 5, for part (d) d = 5 and b = 7.

You can refer to Exercises (1)(a) and (1)(d) for help.

	General	1	2	3	4		10		n
(a)	5n + 1 :	6,	11,	16,	21,	...,	51,	...,	5n + 1
(b)	1n + 5 :	6,	7,	8,	9,	...,	15,	...,	n + 5
(c)	7n + 5 :	12,	19,	26,	33,	...,	75,	...,	7n + 5
(d)	5n + 7 :	12,	17,	22,	27,	...,	57,	...,	5n + 7

Notice: Part (b) could have been written as n + 5.

In (a) and (d) each term is 5 more than its predecessor.

Part (c) has the same first term as part (d). When n = 1, the value of 7n + 5 equals the value of 5n + 7. This is true because multiplying by 1 leads to $7 \cdot 1 + 5 = 5 \cdot 1 + 7$; but these expressions differ from each other whenever $n \neq 1$.

Reminder: The general term is an algebraic expression that is evaluated for selected values of the variable n.

C O M M E N T A R Y

Further Comments About Exploration Exercises

In this course you are being asked to find your way by personal exploration, examination, and Exercise. This is much like learning how to reach a new destination in an area where some locations are familiar and others are strange to you. If you drive to that destination, it is best that you be in the driver's seat: Explore the route with as much guidance as you need, examine where you have been, and then go over that same path until it becomes very familiar to you. The most important element is that you be at the wheel; if someone else drives while you only observe, it is much harder to absorb the directions and reach the same destination alone.

If you examine the notes from this point of view, you will see that the Exploration Exercises provide a general guide. Each one starts at a place that should be familiar to you and outlines a path that leads to the answer for that particular question. If you have difficulty with the Explorations, then use the Solutions that follow; they provide a more detailed map. When a solution is confusing, it is time to ask for personal assistance.

Reminder: The Exercises are designed so that you can ask about the specific points where you were "lost".

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COMMENTARY

Suggestions for Reviewing Recently Studied Mathematics

As a mature student, you will be expected to make decisions about how and what you review. The following suggestions may help.

1. See the following check lists of skills and related study hints. Answer the questions carefully and use your responses to determine what you should review.

2. Use the Solutions to Exercises in a manner appropriate to your needs.

(a) If a particular set of Exercises was difficult for you, examine your attempted answers and compare them with those exhibited by the Solutions. Try to determine the cause of your difficulty:

Did you have trouble understanding the question?

Had you forgotten some basic rule, or did you know the rule without seeing that it would be useful?

Was your difficulty primarily with arithmetic computations?

(b) If you did the Exercises successfully, you can gain from the explanations given with the Solutions. They help you learn why the answers are correct, and thereby establish a base for further learning.

3. Examine the work you did for the Practice Sets, and decide whether you want to use Extra Practice questions for further reinforcement.

4. Diagnostic Tests and Unit Self-Tests, in the Review section, are available for your use; they help you decide what additional study is appropriate.

5. Reference Packages are just that-- items to which you can refer if you want to. No need to memorize; use the references when you choose.

6. Try the Exceptions and Expose Errors questions; they offer a style of review that will help you establish new ways to examine your own work.

In general, the concept of maturity is used here to indicate a student's ability to recognize approaches that contribute to personal success in the study of mathematics. You are given options because there are different ways to learn, different levels of interest, and different amounts of content needed. When reviewing, you can take care of your personal situation by seeking good advice and coming to your own decisions.

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SUMMARY and REVIEW

Ex Post Facto

(Look back at what you have done and act accordingly.)

This course is designed to provide some intuitive experiences with algebra. Look at some of those experiences to see how they are useful in the study of mathematics.

Checklist of Skills

The items on this checklist summarize what you should be able to do after learning the material of this topic. The left hand section asks about your algebraic skills and the right side asks about related computations. Each item is followed by a general study hint that applies to various algebraic situations.

Comment: The questions in the right hand column are not test items; each helps to describe an algebraic skill by asking about a computation or a specific case.

Algebraic Skill	Related Question
1. Can you use the general term of an arithmetic sequence to find each term of that sequence?	What are first ten terms of the sequence 7n + 3?

Look at algebraic generalizations as statements that contain an extensive amount of information in brief form.

2. Can you use a sequence in the form dn to help you find the sequence in the form dn + b?	After you find the terms for 7n, how do you find the related term of 7n + -3 or 7n - 3?

Use something you know, or a previous computation, to help with a new question.

Algebraic Skill	Related Question
3. Can you use the general term of an arithmetic sequence to determine the difference between adjacent terms?	What is the value of d, the common difference between terms, for the sequence described by 7n - 5?

Many algebraic expressions describe a regular pattern; determine that pattern whenever you can.

4.a. Can you find the common difference between terms of a given arithmetic sequence, and use it appropriately in the algebraic expression for the general term?	How do you find the common difference in the sequence 7, 10, 13, 16...? Where do you use this value in the general term dn + b?
b. Can you use the answer to part a, with the first term of a given arithmetic sequence to determine the value of the constant in the general term?	How does 7 relate to the values of b and d in the expression dn + b which represents 7, 10, 13, 16..?

If you have the form for an algebraic generalization, such as dn + b, learn the steps needed to determine the value of each part of that generalization.

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Exceptions **Evaluating Formulae**

Each of the following sets exhibits an applied situation represented by an algebraic expression. For three of the four cases of a given set, there is appropriate and sufficient information to substitute in the equation; one of the cases is an exception. Find the exception.

1. Shipping charges can be represented by the formula $C = dx + f$, where f represents the charge for the first kilogram, d stands for the rate for additional kilograms, and x shows the number of kilograms to be shipped. Which of the following does not have sufficient information for use of this formula?

(a) You know that they charge you \$3 to ship 1 kg, \$5 for 2 kg, and \$11 for 5 kg; what does it cost you to ship an 8 kg package?

(b) Shipping weight of shoes is 1.2 kg per pair. The customer pays a \$1.50 shipping charge for each pair. What is the seller's cost of shipping a 6 kg package?

(c) The rate for shipping large parcels is \$40 per 10 kg, with a minimum charge based upon a 20 kg package. Partial rates are charges based upon units of 1 kg. What is the charge for shipping a 26 kg parcel?

(d) Fragile items have a service charge of \$10 for special handling, and a weight charge of \$3 per kg. What is the charge for shipping a 1 kg fragile parcel?

Exceptions (cont.) Evaluating Formulae

2. The Annual Percentage Rate, APR, for installment payments may be higher than you think because you are regularly reducing the amount you owe. The following formula gives the approximate value of the APR for an installment loan paid monthly.

$APR = \frac{24F}{MN(N+1)}$, where F is the total of all finance charges;
M is the amount of the monthly payment;
N is the number of payments.

Which of the following situations does not give enough information to allow you to compute the approximate APR?

(a) You make monthly payments of $25 each for a loan of $500 and finance charges of $250.

(b) You pay $250 in finance charges for a $500 loan, so the total cost is $750. Monthly payments are all the same.

(c) You make 30 monthly payments of $25 each. This pays off the loan, plus $250 in finance charges.

(d) You make 30 monthly payments to pay off a $500 loan plus $250 finance charge.

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Expose Errors (circle the error) **Evaluating Formulae**

1. Find the area of a trapezoid.

$A = \frac{1}{2}b_1h + \frac{1}{2}b_2h$ or $A = \frac{1}{2}(b_1 + b_2)h$

(a) $b_1 = 16$ cm, $b_2 = 8$ cm, $h = 4$ cm

$A = \frac{1}{2}(8 + 4)4 = 6(4) = 24$ cm^2

(b) $b_1 = 15.8$ cm, $b_2 = 7.5$ cm, $h = 3.8$ cm

$A = \frac{1}{2}(15.8 + 7.5)3.8 = 23.3(1.9) = 44.3$ cm^2

(c) $b = 15\frac{3}{4}$ cm, $b = 7\frac{1}{2}$ cm, $h = 3\frac{3}{4}$ cm

$A = (\frac{63}{4} + \frac{15}{2})\frac{15}{4} = \frac{93}{4}(\frac{15}{4}) = \frac{1395}{16} = 87\frac{3}{16}$

$A = 87$ cm^2 approx.

2. Find the new price on an item which has a discount from the manufacturer's suggested price.

$N = S - D$ and $D = rS$, so $N = S - rS$.

(a) $S = \$78$, $D = 25\%$ of S

$N = 0.25(78) = \$19.50$

(b) $S = \$79.95$, $D = 20\%$

$N = 79.95 - 0.20(\$79.95)$

$N = 0.80(79.95) = \$63.96$

(c) $S = \$89.50$, $r_1 = 20\%$, $r_2 = 10\%$, where there is a second discount, $D_2 = r_2N_1$ and the newest price is $N_2 = N_1 - D_2$.

$N = 89.50 - 0.20(89.50) - 0.10(89.50)$

$= 89.50 - 17.90 - 8.95$

$N = \$63.65$

Expose Errors (cont.) Evaluating Formulae

3. Find the total interest received on two different investments.

$I = P_2 r_1 t_1 1 + P_2 r_2 t_2$, simple interest.

(a) $P_1 = \$1200$, $r_1 = 6\%$ per year, $P_2 = \$1600$,

$r_2 = 16\%$ per year,

$t_1 = t_2 = 6$ months $= \frac{1}{2}$ year

$I = (1200)(6)(\frac{1}{2}) + (1600)(16)(\frac{1}{2})$

$= (600)(6) + (800)(8) = 3600 + 4800$

$I = \$8400$

(b) $P_1 = \$1250$, r_2 5.86% per year, $P_2 = \$1650$,

$r_2 = 15.96\%$ per year, $t_1 = t_2 = 0.5$ year

$I = 1250(0.586)(0.5) + 1650(0.1596)(0.5)$

$I = 366.25 + 131.67 = \$497.92$

(c) $P_1 = \$1200$, $r_1 = 5\frac{3}{4}\%$, $P_2 = \$1650$, $r_2 = 15\frac{3}{4}\%$

$t_1 = t_2 = \frac{1}{2}$ year

$I = 1200(\frac{23}{400})(\frac{1}{2}) + 1600(\frac{63}{400})(\frac{1}{2})$

$= \frac{3}{2}(23) + \frac{4}{2}(63)$

$= \frac{69 + 252}{2}$

$I = \$110.50$

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Annotated Answers to Exceptions

Evaluating Formulae

For your interest, answers are given. Nevertheless, you should remember that this work asks you to look for the question that contains insufficient information, i.e, to focus upon finding the exception in a set of similar questions. The exceptions are marked (xx) and explained. Comments provide an expanded discussion in order to improve your understanding of applied problems.

1. Shipping charges: $C = dx + f$

(a) \$17 for 8 kg (c) \$104 for 26 kg (d) \$13 for 1 kg;

Comments: In part (d) there are no additional kilograms, so the value of x is 0 here; $f = 13$.

(XX)(b) You are asked for the seller's cost, but are given information only about the charge to the customer.

2. $APR = \dfrac{24F}{MN(N + 1)}$

APR for (a) (c) and (d) is approximately 26%.

Comments: In each case $F = \$250$, $M = \$30$, $N = 25$, and the loan is \$500.

(XX)(b) The total cost is the same here, but the number of payments cannot be determined. Therefore you cannot determine the amount of each payment or the APR.

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Annotated Answers to Expose Errors

Evaluating Formulae

1. $A = \frac{1}{2}(b_1 + b_2)h$

(a) $A \neq \frac{1}{2}(8 + 4)4$; incorrect substitutions here, so

$A \neq 24\ cm^2$

$A = \frac{1}{2}(16 + 8)4 = 48\ cm^2$

(b) All computations are correct. The answer, 44.3 cm^2, is correct; it is given to the nearest 0.1 cm^2

Notice: Values in 1(a) are good estimates for 1(b), (c); use them for a quick check to see if other answers are reasonable.

(c) $A \neq (\frac{63}{4} + \frac{15}{2})\frac{15}{4}$, therefore $A \neq 87\ cm^2$

$A = \frac{1}{2}(\frac{63}{4} + \frac{15}{2})\frac{15}{4} = \frac{1}{2}(\frac{93}{4})(\frac{15}{4}) = 44\ cm^2$ approx.

2. $N = S - D$ or $N = S - rS$.

(a) $N \neq 0.25(78)$, so $N \neq \$19.50$.

$D = 0.25(78) = 19.50$, so $N = 78 - 19.50 = \$58.50$.

(b) This answer is correct; $N = \$63.96$.

Notice: The new price is 80% of the original price.

(c) $N_2 \neq 89.50 - 0.20(89.50) - 0.10(89.50)$

$N_1 = 89.50 - 17.90 = 71.60$ after first discount.

Therefore, $N_2 = 71.60 - 0.10(71.60) = \$64.44$

Comment: $89.50 - 17.90 - 8.95 \neq 63.65$. It would have been 62.65 if the given substitutions were correct, but \$62.65 is not the correct answer.

In other words, finding a computational error is not enough if you substituted the wrong values.

3. $I = P_1r_1t_1 + P_2r_2t_2$

(a) $I \neq (1200)(6)(\frac{1}{2}) + 1600(16)(\frac{1}{2})$; check percent.

Also examine computations: $(1600(16)(\frac{1}{2}) \neq (800)(8)$,

and $(800)(8) \neq 4800$.

$I \neq \$8400$

$$I = (1200)(0.06)(\tfrac{1}{2}) + (1600)(0.16)(\tfrac{1}{2})$$

$$= \$36.00 + \$128.00 = \$164.00$$

(b) $I \neq \$497.92$ because $I \neq 366.25 + 131.67$

$I \neq 1250(0.586)(0.5) + 1650(0.1596)(0.5)$

$$I = 1250(0.0586)(0.5) + 1650(0.1596)(0.5)$$

$$= 36.63 + 131.67 = \$168.30$$

Notice: The interest at 5.86% for \$1250 must be much less than the interest at 15.96% for \$1650.

5.86% = 0.0586 and 15.96% = 0.1596

(c) $I \neq 1200(\frac{23}{400})(\frac{1}{2}) + 1600(\frac{63}{400})(\frac{1}{2})$

$$I = 1200(\tfrac{23}{400})(\tfrac{1}{2}) + 1650(\tfrac{63}{400})(\tfrac{1}{2})$$

$$= \tfrac{3}{2}(23) + \tfrac{165}{80}(63) = \frac{2760 + 10395}{80} = \frac{13155}{80}$$

$$= \$164.44$$

Because the substitution was incorrect, the computations are irrelevant. Nevertheless, examine them:

$\frac{69 + 252}{2} \neq 110.50$

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STUDENT RESPONSES TO THE LEARNING APPROACH IN INTRODUCTORY ALGEBRA FOR MATURE STUDENTS

At the end of the first month of class, students were asked to respond to the COMMENTARY that discussed the approach used in the class. (See page 8 herein). Some sample responses follow; selected from the 80% who indicated agreement.

1. In the past, I've always expected teachers to "give me" an education, while I looked on. This time...I feel more personal responsibility for my own education.

2. I have found (the notes) very helpful when working at home. (They) explain in detail how to work a problem and even (have) you look back and review problems that are similar to it.

3. I have learned that it works best to work out something yourself rather than watch someone do it.

4. The (notes) explain and show how to do the problems, making the route to finding the answer much easier.

5. I found the class structure to be very hard for me to deal with in the beginning. But now I have seen that it is up to me and no one else to make it work for me, so I have accepted the structure and am working with it.

6. It is good to have the answers ... with an explanation of how the problem was done so that you can check your work and make sure you are doing it correctly.

7. I believe that this commentary is an adequate description of how one could view the class.

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PREFACE: TO THE STUDENT

This course is for mature students who realize that basic algebraic skills are needed for studying and understanding almost any professional or paraprofessional field. INTRODUCTORY ALGEBRA presupposes no knowledge of algebraic procedures, and yet you will be encouraged to work algebraically in an intuitive way from the onset of the course.

Four essential skills needed by the beginning student in algebra are emphasized throughout:

1. The ability to make sense of mathematical writing, and focus upon the meaning and requirements of a question;
2. The ability to use fundamental ideas of arithmetic in new settings;
3. The ability to use the proper order of operations;
4. The ability to recognize differences among arithmetic or algebraic expressions that are superficially similar.

These skills are developed in contexts that are new to the average reader, so that you, as an adult, aren't faced with a dreary reminder of adolescent studies. For example, you will get extensive practice with arithmetic skills during your study of sequences. At the same time, you will be challenged to find or focus upon the general rule for a sequence, while you apply simple algorithms (rules) using basic skills.

In case your intuition or experience isn't sufficient to meet the challenge, there is immediate support in the accom-

panying solutions. In many ways, the Solution sections replace the usual exposition of a mathematical text, with one very important difference: before you read the Solutions, the Exercises will make you examine the questions. An answer means much more to someone who has tried an Exercise than it does as part of a routine question/answer exposition. Therefore, you are encouraged to participate actively, and to try Exercises at a simple level, early in the development of each algebraic topic.

The intent is to make both new vocabulary and mathematical explanations more palatable and intellectually nourishing. Your potential reading problems as a beginning student are recognized. They are met by introducing new vocabulary and technical explanations in a context where you can be successful, even when you may not fully understand the vocabulary or explanation at first. Use your completed work to help you with new words and ideas; do the computations, then re-read the explanations. In addition you will find help in Vocabulary and Notation Reference Packages related to each topic.

As mentioned previously, you will find early opportunities to sharpen arithmetic skills which are needed later in the course. Usually, they are skills which are required even by the owner of hand calculator, e.g., using the proper order of operations, and the selection of common denominators. You will practice these skills in settings which have other purposes as well, e.g., when looking for equivalence or when examining mathematical differences among algebraic statements that look similar.

The introduction to algebraic ideas at the beginning of the course is an intellectual warm-up with questions that most students can do successfully. Furthermore, you are encouraged to know when you are right, e.g., to check your answers by simple substitution. Success at this level sets the stage with a feeling of new accomplishment in a new course. It helps most people feel comfortable with mathematics and ready to explore ideas that are new to them.

The rest of the course continues in the same style; new techniques are explained in a comfortable setting, using familiar skills. The Exploration Exercises are at the heart of this development and give most students a successful introductory experience with each new concept. (See **COMMENTARIES:** How to Use Exploration Exercises, and Further Comments About Exploration Exercises).

In the past, most students who have read similar material eventually wanted to know why a given rule was used. This attitude is developed by the gradual explanation of reasons for algebraic techniques, and the soft light cast on some fundamental properties of algebra. In short, progress through the course can bring you from the naive, but essential, guess-and-check method to a more formal approach that probes the mystery in the refrain, "find x."